APPLICATION OF A SHADOW INSTRUMENT FOR DETERMINING TURBULENCE CHARACTERISTICS

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The development of experimental investigations in the field of turbulence studies is accompanied by improvement of the methods used in studying the medium. The optical methods are being more widely used at the present time for the study of fine-scale turbulence [1-3].

The interest in the optical methods is explained by their great capabilities. The optical instruments have practically instantaneous response, high sensitivity, and high spatial and frequency resolution. It is significant that the optical methods do not have any disturbing action on the medium in the investigated volume.

One of the basic optical methods used to study turbulence is the shadow method. In this method use is made of the deflection of light rays passing through an optically nonuniform medium. The presence of optical nonhomogeneities in the turbulent medium is associated with pulsations of the density or the refractivity of the medium. These changes are due to pulsations of such characteristics of the medium as temperature, velocity, pressure, additive concentration, and so on.

Photographic or photoelectric data recording is used in the shadow instruments. The shadowgraphs are usually used to study the structure of turbulent flows in conducting aerodynamic and plasma studies [3, 4].

We shall examine the possibility of using shadow instruments with photoelectric data recording to measure the characteristics of a turbulent medium. We take as the example the optical scheme of the Topler method (Fig. 1). Let the light diaphragm M in the form of a circular opening of radius r_0, uniformly illuminated from the source S, be located in the focal plane F_1 of the collimating objective O_1. The objective O_2 projects the image of the light diaphragm in the plane F_2, in which there is located the screen N (shadow diaphragm) in the form of a circle of the same radius r_0. We locate the photoelectric receiver Q behind the shadow diaphragm to record the light flux. The test medium is located between the objectives O_1 and O_2.

With regard to information transformation the optical instrument using the described scheme can be represented as a system consisting of four basic elements, shown in Fig. 2, where the elements 1, ..., 4 correspond to

$$\begin{array}{cccc} 1 & 2 & 3 & 4 \\ \langle \varepsilon^2 \rangle = k_1 \langle n'^2 \rangle, & \rho_* = k_2 \langle \varepsilon^2 \rangle^{1/2}, & \Delta\Phi = k_3 \rho_*, & u = k_4 \Delta\Phi \end{array}$$

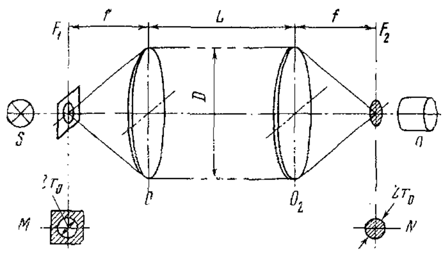

Fig. 1

The basic element in this system is the investigated volume of the medium. The fluctuations of the propagation angle of the light rays passing through the investigated volume of the turbulent medium carry information on the properties of the turbulence proper. The most important turbulence characteristic [5] is the structure function, defined, for example, for the light refractivity pulsation field in the medium as [6]

$$D_n(r) = \langle n'^2 \rangle = \langle (n_1 - n_2)^2 \rangle = C_n^2 r^{2/3} \tag{1}$$

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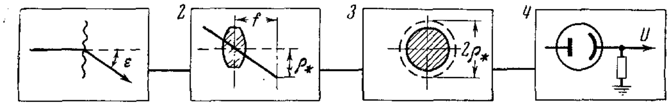

Fig. 2

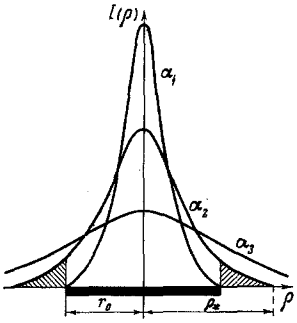

Fig. 3

Here C_n^2 is the structure constant of the medium, r is the distance between two points with local index-of-refraction values n_1 and n_2.

The relation (1) is valid for the inertial turbulence region, i.e., with satisfaction of the condition

$$l \ll r \ll L_0$$

Here L_0 is the external turbulence scale; l is the internal turbulence scale, commensurate with the radius of correlation of the optical nonhomogeneities.

The structure function $\langle n'^2 \rangle$ characterizes the "intensity" of the index-of-refraction pulsations in the medium with scales not exceeding r in order of magnitude. To evaluate the pulsation intensity we also use [7] the quantity $\langle n'^2 \rangle^{1/2}$, which is also called the structure function.

The structure function of the index-of-refraction field of the medium can be related with the fluctuations of the light ray deflection angle. For the case in which the distance L traveled by the light wave in the turbulent medium satisfies the condition $L \ll l^2/\lambda$, where λ is the wavelength of the light, we can use the approximation of geometric optics [8]. According to [9] the following relation holds for the isotropic medium

$$\langle \varepsilon^2 \rangle = {}^2/_3 \langle n'^2 \rangle L / l \tag{2}$$

Here $\langle \varepsilon^2 \rangle$ is the dispersion of the light ray deflection angle at the exit from the investigated volume.

We see from (2) that by measuring the fluctuations of the light ray angle of arrival we can determine the most important turbulence characteristic – the structure function or the structure constant of the medium.

It is significant that the light ray deflection angle fluctuation spectrum $F_\varepsilon(\varkappa)$ represents the turbulence spectrum $\Phi_n(\varkappa)$. According to [10], for an isotropic index-of-refraction pulsation field

$$F_\varepsilon(\varkappa) \sim \pi k^2 L \Phi_n(\varkappa), \qquad k = 2\pi/\lambda \quad (\varkappa \text{ is the spacewise frequency}) \tag{3}$$

One of the characteristics of light source image transfer through an isotropic turbulent medium is the axial symmetry of the scattered light angular distribution [11]. This circumstance must lead to a situation in which the image of the circular light source obtained in the focal plane of the objective of the subject instrument "diffuses." The dimensions of the diffuse spot are determined both by the intensity of the turbulence in the investigated volume and by the parameters of the objective O_2.

The instrument objective O_2 should be considered the second element in the information conversion system. The radius of the light spot in the objective focal plane can be found from the expression

$$\rho_* = \langle \varepsilon^2 \rangle^{1/2} f \tag{4}$$

Here f is the focal length of the objective.

For a given instrument the transfer coefficient of the objective is a constant quantity.

For the subsequent analysis it is important to know how the illumination is distributed in the light spot obtained in the objective focal plane. The form of this relation is determined by the intensity of the index-of-refraction pulsations in the medium. We differentiate the cases of small and large index-of-refraction pulsations in the medium [12]. To do this we use the parameter

$$\alpha = \sqrt{\pi} \langle n'^2 \rangle \frac{4\pi^2}{\lambda^2} l L \tag{5}$$

When $\alpha >> 1$ and the instrument objective diameter $D >> l$ the illumination in the image of the point light source decreases with distance from the focus in accordance with the expression [12]

$$I(\rho) = I_0 [\exp(-\beta\rho^2)], \quad I_0 = \frac{\pi l D^2}{\alpha\lambda^2 f^2} A_0^2, \quad \beta = \frac{1}{\alpha}\left(\frac{\pi l}{\lambda f}\right)^2 \tag{6}$$

Here I_0 is the illumination at the focus, ρ is the distance from the focus, β is the attenuation factor, A_0 is the light wave amplitude.

The effective radius of the light spot can be considered to be

$$\rho_* = \frac{1}{\sqrt{\beta}} = \sqrt{\alpha}\,\frac{\lambda f}{\pi l} \tag{7}$$

since at the distance ρ_* the illumination in the image of the light source becomes negligibly small (here, as before,we neglect diffraction effects).

It is not difficult to see that the expressions (4) and (7) for ρ_* agree with one another. Since in the subject Topler instrument the light spot in the focal plane of the objective O_2 is blocked by the shadow diaphragm of radius r_0, only that part $\Delta\Phi$ of the light flux from the source which is not vignetted by the diaphragm strikes the photoreceiver located behind the shadow diaphragm.

The "light spot-shadow diaphragm" can be considered as the third element in the information conversion circuit in the optical instrument. Let us establish the form of the dependence of the differential light flux $\Delta\Phi$ in the instrument on the intensity of the index-of-refraction pulsations in the investigated medium $\langle n'^2 \rangle^{1/2}$. Figure 3 shows the variation of the illumination in the image of the point source obtained in the plane of the instrument shadow diaphragm with change of the pulsation parameter α in the medium $(\alpha_1 < \alpha_2 < \alpha_3)$.

To determine the value of $\Delta\Phi$ we use (6). Converting to polar coordinates, we have

$$\Delta\Phi = \int_0^{2\pi}\int_{r_0}^{\rho_*} I_0(\theta, \rho)\,\rho d\rho\, d\theta = \int_{r_0}^{\rho_*} I_0[\exp(-\beta\rho^2)]\,\rho d\rho \int_0^{2\pi} d\theta$$

Hence

$$\Delta\Phi = \frac{\pi I_0}{\beta}[\exp(-\beta r_0^2) - \exp(-\beta\rho_*^2)] \tag{8}$$

After substituting (6) and (7) into (8), we obtain

$$\Delta\Phi = D^2 A_0^2 \left[\exp\left(-\frac{r_0^2 l}{4\sqrt{\pi}\,L f^2}\frac{1}{\langle n'^2 \rangle}\right) - \frac{1}{e}\right] \tag{9}$$

Evaluation of (9) for known instrument parameters shows that there is an approximately linear connection between the quantities $\Delta\Phi$ and $\langle n'^2 \rangle^{1/2}$ in the range of small changes of $\langle n'^2 \rangle^{1/2}$.

In (9) there is one undefined quantity – this is the optical nonhomogeneity correlation scale l. Usually the value of l is specified (see, for example, [13]) in practical calculations of the parameters of the images of objects observed through a turbulent medium. To specify this value we use either the results of corresponding measurements of the scale l or we calculate this quantity from the relations which are known in aerodynamics, for example for the velocity field

$$l = \sqrt{\nu^3/\varepsilon_0} \tag{10}$$

Here ν is the kinematic viscosity of the medium, ε_0 is the rate of kinetic energy dissipation in the medium.

Relation (9) is valid for the case of a point light source with satisfaction of the inequality $r_0 \leq \rho_*$. However, as a rule in actual instruments the light source must be considered a source of finite dimensions. In this case the radius of the light spot in the plane of the shadow diaphragm can be written in the form

$$\rho_k = r_0 + \rho_* \tag{11}$$

where r_0 is the radius of the light diaphragm in the radiator.

Analysis of the light flux distribution in the plane of the shadow diaphragm from the viewpoint of determining the differential flux $\Delta\Phi$ for different pulsations of $\langle n'^2 \rangle^{1/2}$ in the medium shows that the range of linearity of the characteristic $\Delta\Phi = \varphi(\langle n'^2 \rangle^{1/2})$ for a light source of finite dimensions is significantly larger than is the case for the point source.

The fourth element in the optical instrument, just as in an information transformation system, can be considered the photoreceiver, which is usually used in the linear regime. Thus, linearity with respect to the quantity $\langle n'^2 \rangle^{1/2}$ is maintained in all the instrument elements.

LITERATURE CITED

1. N. F. Derevyanko and A. M. Trokhan, "Optical methods for studying turbulence," PMTF (J. Appl. Mech. and Tech. Phys.), 9, No. 4 (1968).
2. D. Holder and R. North, Shadow Methods in Aerodynamics [Russian translation], Mir, Moscow (1966).
3. L. A. Dushin and O. S. Pavlichenko, Plasma Study Using Lasers [in Russian], Atomizdat, Moscow (1968).
4. M. S. Uberoi and L. S. G. Kovasznay, "Analysis of turbulent density fluctuations by the shadow method," J. Appl. Phys., 26, No. 1 (1955).
5. A. N. Kolmogorov, "Local turbulence structure in incompressible fluid at very high Reynolds numbers," Dokl. AN SSSR, 30, No. 4 (1941).
6. A. M. Obukhov, "Structure of temperature field in turbulent flow," Izv. AN SSSR, Ser. Geogr. i Geofiz., 13, No. 1 (1949).
7. A. S. Monin and A. M. Yaglom, Statistical Hydromechanics [in Russian], Part 2, Nauka, Moscow (1967).
8. A. S. Gurvich, "Determining turbulence characteristics from light propagation experiments," Izv. AN SSSR, Ser. Fiz. Atmosfery i Okeana, 4, No. 2 (1968).
9. M. M. Weiner, "Atmospheric turbulence in optical surveillance systems," Appl. Optics, 6, No. 11 (1967).
10. V. I. Tatarskii, Wave Propagation in a Turbulent Atmosphere [in Russian], Nauka, Moscow (1967).
11. M. V. Kabanov, "Optical transfer function for scattering media," Izv. AN SSSR, Ser. Fiz. Atmosfery i Okeana," 4, No. 8 (1968).
12. L. A. Chernov, Wave Propagation in Medium with Random Nonhomogeneities [in Russian], Izd-vo AN SSSR (1958).
13. I. G. Kolchinskii, Optical Instability of Earth's Atmosphere from Star Observations [in Russian], Naukova Dumka, Kiev (1967).